Beam shifting of an anisotropic negative refractive medium

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We rigorously derive the formulas of the total electric and magnetic fields in a slab of anisotropic negative refractive medium by use of Maxwell's equations. A beam shifting is exhibited from a Gaussian beam for an anisotropic negative refractive slab. Our results indicate that there exist two possible (positive or negative) beam shifting for an anisotropic negative refractive medium due to the special anisotropic properties, which is distinct from the case of an isotropic negative refractive slab. Physical insights are also presented.

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I. INTRODUCTION

Recently, extensive research efforts have centered on composite materials with simultaneously negative permittivity and permeability [1–10]. Such media are usually called left-handed media because the relationship between the electric field, the magnetic intensity, and the direction of propagation of electromagnetic waves traveling through them obey a left-handed rule rather than the conventional right-handed rule, which was first introduced by Veselago [11]. A so-called left-handed material results in many unusual properties, such as anomalous negative refractive index, Doppler shift, and Cherenkov radiation [11]. If a beam impinges at the interface between two media, one of which exhibits a negative refractive index and the other a positive refractive index, the transmitted wave will lie in the same side of the surface normal line as the incident wave, rather than the opposite side. This character is applied to demonstrate whether the material is left handed or not based on a prism experiment [12]. There is another way to identify whether the material is a left-handed material or not by observing the beam shifting of a Gaussian beam [13]. Some brief discussions of beam shifting have been extended toward the case of isotropic negative refractive media. It has been found that a dramatic negative lateral shift is observed when a slab is an isotropic material with both negative permittivity and permeability [14,15]. With the exception of some brief discussions, the emphasis of theoretical studies has focused on the case where the negative refractive material is isotropic. In practice, the structures (interlacing wires and split-ring resonators) investigated in experiments are strongly anisotropic. The positive and negative refracting layers of anisotropic media have combined to substitute conventional spatial filtering [16,17]. Compared with isotropic negative refractive media, anisotropic negative refractive media may exhibit distinct differences in physical properties, such as anomalous reflection and refraction at the interface of isotropic regular media and uniaxially anisotropic negative refractive media [18]; especially the directions of the group and phase velocities in an anisotropic medium, however, are not fixed but rather vary with the direction of propagation in the material with respect to the principal axes [18]. It has been found that reflection shows frequency-selective total oblique transmission that is distinct from the Brewster effect at a planar dispersive negative index interface [19]. Due to the anisotropic characters of left-handed material in experiments, we should examine whether it will experience negative beam lateral shifting or not. It is our motivation to present a detail analysis of the beam shifting from a Gaussian beam for an anisotropic negative refractive slab. Our calculations show that there exist two possible (positive or negative) beam shiftings for an anisotropic negative index medium due to the anisotropic properties, which is distinct from the case of an isotropic negative refractive slab. We hope to generalize the situation in order to understand the anisotropic negative refractive medium at greater depth.

II. DISTRIBUTION OF THE ELECTRIC AND MAGNETIC FIELDS

We consider an anisotropic negative refractive slab which separates two semi-infinite media. Two semi-infinite media are isotropic positive refractive material, one of which is characterized by permittivity ε_1 and permeability μ_1 (region 1) and the other is ε_2 and μ_2 (region 2). The anisotropic negative refractive medium is denoted as $\hat{\varepsilon}$ and $\hat{\mu}$, and the thickness of the slab is d. The geometrical structure considered in this paper is shown in Fig. 1. Here we assume that anisotropic permittivity and permeability tensors can be simultaneously diagonalizable [16]; then the permittivity $\hat{\varepsilon}$ and

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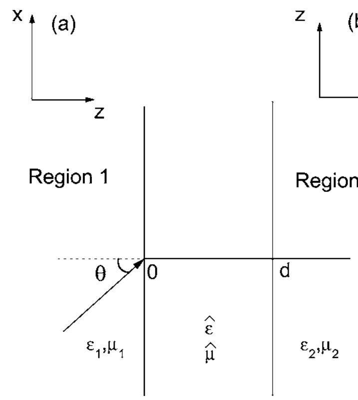

FIG. 1. Geometrical structure of the problem.

permeability $\hat{\mu}$ of the anisotropic negative refractive medium are denoted as the tensor forms

$$\hat{\varepsilon}=\begin{pmatrix}\varepsilon_x & 0 & 0\\ 0 & \varepsilon_y & 0\\ 0 & 0 & \varepsilon_z\end{pmatrix}, \tag{1}$$

$$\hat{\mu}=\begin{pmatrix}\mu_x & 0 & 0\\ 0 & \mu_y & 0\\ 0 & 0 & \mu_z.\end{pmatrix}. \tag{2}$$

Not all principal elements possess the same sign [16]. TE and TM modes can be considered separately. We focus our attention on TE waves in this paper. A similar disposal can be carried out for TM waves. First, we consider the case that the interface is at the x-y plane. The electric field from region 1 is expressed as

$$E_{1y}=\int_{-\infty}^{+\infty} dk_x \phi(k_x) e^{ik_x x+ik_{1z}z}, \tag{3}$$

where $\phi=(g/2\sqrt{\pi})e^{-g^2(k_x-k_{ix})^2/4}$ is the Gaussian spectrum, which carries the information on the shape of the footprint centered at $x=0$, $z=0$. $k_{ix}=(\omega/c)\sqrt{\varepsilon_1\mu_1}\sin\theta$ determines the angle of the center of the incident beam, ω is the angular frequency of the incident beam, and c stands for the velocity of light in the vacuum. Note that the component of the wave vector along the x direction is the same in all regions, whereas the component of the wave vector along the z direction is discontinuous. The most important thing in anisotropic negative refractive material is how to choose the sign of the square roots of the wave vector. The phase velocity and Poynting vector are antiparallel strictly for isotropic negative refractive index material, which makes it easy to determine the sign of the wave vector. However, the directions of the group and phase velocities in an anisotropic medium are not fixed but rather vary with the direction of propagation. The signs of the components of the permittivity and permeability of anisotropic negative refractive material, moreover, will have great influence on the choice of square roots of the wave vector. We would like to stress the fact that the choice of the sign of the wave vector must ensure a Poynting vector inside the anisotropic medium to point away from the interface between the incident and anisotropic media. For the media in regions 1 and 2, the explicit expressions of the wave vector along the z direction are easily obtained with $k_{1z}=\sqrt{(\omega^2/c^2)\varepsilon_1\mu_1-k_x^2}$ and $k_{2z}=\sqrt{(\omega^2/c^2)\varepsilon_2\mu_2-k_x^2}$. To further clarify the sign of the wave vector, we give the Poynting vectors by

$$\vec{S}_1=\frac{|E_0|^2}{2\omega}\left[\frac{k_x}{\mu_1}\vec{e}_x+\frac{k_{1z}}{\mu_1}\vec{e}_z\right]$$

in region 1 and

$$\vec{S}=\frac{|E_0|^2}{2\omega}\left[\frac{k_x}{\mu_z}\vec{e}_x+\frac{k_z}{\mu_x}\vec{e}_z\right]$$

in an anisotropic slab for a given k_x, where $S_{1z}>0$. Now the continuity of the signs of the Poynting vector along the normal direction requires that $\vec{e}_z\cdot\vec{S}>0$—i.e., k_z and μ_x must have the same sign—whereas the tangential direction of the Poynting vector will determine whether the refraction is anomalous (negative refraction) or not. Because both components of the Poynting vector are positive from the incident medium, when μ_x is positive we must choose $k_z=\sqrt{(\omega^2/c^2)\varepsilon_y\mu_x-(\mu_x/\mu_z)k_x^2}$ to ensure the component of the Poynting vector along the z direction to be positive. On the contrary, $k_z=-\sqrt{(\omega^2/c^2)\varepsilon_y\mu_x-(\mu_x/\mu_z)k_x^2}$ if μ_x is negative for an anisotropic negative refractive medium. According to Maxwell's equations, the magnetic field in region 1 can be obtained as

$$H_{1x}=\int_{-\infty}^{+\infty} dk_x \phi(k_x)\left[-\frac{k_{1z}}{\omega\mu_1}e^{ik_{1z}z}+R\frac{k_{1z}}{\omega\mu_1}e^{-ik_{1z}z}\right]e^{ik_x x}, \tag{4}$$

$$H_{1z}=\int_{-\infty}^{+\infty} dk_x \phi(k_x)\left[\frac{k_x}{\omega\mu_1}e^{ik_{1z}z}+R\frac{k_x}{\omega\mu_1}e^{-ik_{1z}z}\right]e^{ik_x x}. \tag{5}$$

In the region where the anisotropic negative refractive medium occupies, the electric and magnetic fields are expressed as

$$E_y=\int_{-\infty}^{+\infty} dk_x \phi(k_x)[A_2 e^{ik_z z}+B_2 e^{-ik_z z}]e^{ik_x x}, \tag{6}$$

$$H_x=\int_{-\infty}^{+\infty} dk_x \phi(k_x)\left[-A_2\frac{k_z}{\omega\mu_x}e^{ik_z z}+B_2\frac{k_z}{\omega\mu_x}e^{-ik_z z}\right]e^{ik_x x}, \tag{7}$$

$$H_z=\int_{-\infty}^{+\infty} dk_x \phi(k_x)\left[\frac{k_x}{\omega\mu_z}A_2 e^{ik_z z}+\frac{k_x}{\omega\mu_z}B_2 e^{-ik_z z}\right]e^{ik_x x}. \tag{8}$$

In region 2, the electric and magnetic fields are expressed as

$$E_{2y}=\int_{-\infty}^{+\infty} dk_x \phi(k_x) T e^{ik_x x+ik_{2z}z}, \tag{9}$$

$$H_{2x} = \int_{-\infty}^{+\infty} dk_x \phi(k_x) \left[-T \frac{k_{2z}}{\omega\mu_2} e^{ik_{2z}z} \right] e^{ik_x x}, \tag{10}$$

$$H_{2z} = \int_{-\infty}^{+\infty} dk_x \phi(k_x) \left[\frac{k_x}{\omega\mu_2} T e^{ik_{2z}z} \right] e^{ik_x x}. \tag{11}$$

R and T are reflection and transmission coefficients, respectively, and A_2 and B_2 are two unknown constants, embodying the strength of the electric and magnetic fields. R, T, A_2, and B_2 are to be determined from the boundary conditions at $z = 0, d$. The solutions from the boundary conditions are as follows:

$$R = \frac{R_{1x} + R_{x2} e^{2ik_z d}}{1 + R_{1x} R_{x2} e^{2ik_z d}}, \tag{12}$$

$$T = \frac{4e^{i(k_z - k_{2z})d}}{(1 + P_{1x})(1 + P_{x2})[1 + R_{1x} R_{x2} e^{2ik_z d}]}, \tag{13}$$

$$A_2 = \frac{2}{(1 + P_{1x})[1 + R_{1x} R_{x2} e^{2ik_z d}]}, \tag{14}$$

$$B_2 = \frac{2e^{2ik_z d} R_{x2}}{(1 + P_{1x})[1 + R_{1x} R_{x2} e^{2ik_z d}]}, \tag{15}$$

where $P_{1x} = \mu_1 k_z / \mu_x k_{1z}$, $P_{x2} = \mu_x k_{2z} / \mu_2 k_z$, $R_{1x} = (1 - P_{1x})/(1 + P_{1x})$, and $R_{x2} = (1 - P_{x2})/(1 + P_{x2})$. Thus the electric and magnetic fields are determined explicitly in three regions.

Next, due to the anisotropic character of negative refractive medium, there exists another case in which the interface is at the y-z plane. For the present case, we would like to note that $\phi = (g/2\sqrt{\pi}) e^{-g^2(k_z - k_{iz})^2/4}$ is the Gaussian spectrum, where $k_{iz} = (\omega/c)\sqrt{\varepsilon_1 \mu_1} \sin\theta$ determines the angle of the center of the incident beam and the component of the wave vector along the z direction is the same in all regions, whereas the component of the wave vector along the x direction is discontinuous. Similarly, special care must be taken in determining the sign of the wave vector along the x direction. We still need to obey the physics requirement that the Poynting vector inside anisotropic medium flow away from the interface between the incident medium and anisotropic negative refractive medium. We show the explicit expressions of the wave vector along the x direction in regions 1 and 2: $k_{1x} = \sqrt{(\omega^2/c^2)\varepsilon_1\mu_1 - k_z^2}$ and $k_{2x} = \sqrt{(\omega^2/c^2)\varepsilon_2\mu_2 - k_z^2}$. The Poynting vectors of region 1 and an anisotropic slab can also derived as

$$\vec{S}_1 = \frac{|E_0|^2}{2\omega} \left[\frac{k_{1x}}{\mu_1} \vec{e}_x + \frac{k_z}{\mu_1} \vec{e}_z \right]$$

and

$$\vec{S} = \frac{|E_0|^2}{2\omega} \left[\frac{k_x}{\mu_z} \vec{e}_x + \frac{k_z}{\mu_x} \vec{e}_z \right],$$

respectively. Now the normal direction is the direction of the x axis when the interface is at the y-z plane. So the signs of the Poynting vector along the x axis are continuous. $S_x [= (|E_0|^2/2\omega) k_x/\mu_z]$ must larger than zero due to $S_{1x} > 0$, which means that k_x and μ_z must have the same sign. Therefore, for an anisotropic medium, $k_x = \sqrt{(\omega^2/c^2)\varepsilon_y\mu_z - (\mu_z/\mu_x)k_z^2}$ if μ_z is positive and $k_x = -\sqrt{(\omega^2/c^2)\varepsilon_y\mu_z - (\mu_z/\mu_x)k_z^2}$ if μ_z is negative, which ensure the component of the Poynting vector along the x direction to be positive. Then we list their expressions of the electric and magnetic fields in different regions:

$$E_{1y} = \int_{-\infty}^{+\infty} dk_z \phi(k_z) e^{ik_z z + ik_{1x} x},$$

$$H_{1x} = \int_{-\infty}^{+\infty} dk_z \phi(k_z) \left[-\frac{k_z}{\omega\mu_1} e^{ik_{1x}x} - R \frac{k_z}{\omega\mu_1} e^{-ik_{1x}x} \right] e^{ik_z z},$$

$$H_{1z} = \int_{-\infty}^{+\infty} dk_z \phi(k_z) \left[\frac{k_{1x}}{\omega\mu_1} e^{ik_{1x}x} - R \frac{k_{1x}}{\omega\mu_1} e^{-ik_{1x}x} \right] e^{ik_z z}, \tag{16}$$

in region 1,

$$E_y = \int_{-\infty}^{+\infty} dk_z \phi(k_z) [A_2 e^{ik_x x} + B_2 e^{-ik_x x}] e^{ik_z z},$$

$$H_x = \int_{-\infty}^{+\infty} dk_z \phi(k_z) \left[-A_2 \frac{k_z}{\omega\mu_x} e^{ik_x x} - B_2 \frac{k_z}{\omega\mu_x} e^{-ik_x x} \right] e^{ik_z z},$$

$$H_z = \int_{-\infty}^{+\infty} dk_z \phi(k_z) \left[A_2 \frac{k_x}{\omega\mu_z} e^{ik_x x} - B_2 \frac{k_x}{\omega\mu_z} e^{-ik_x x} \right] e^{ik_z z}, \tag{17}$$

in the region which the anisotropic negative refractive medium occupies, and

$$E_{2y} = \int_{-\infty}^{+\infty} dk_z \phi(k_z) T e^{ik_{2x}x + ik_z z},$$

$$H_{2x} = \int_{-\infty}^{+\infty} dk_z \phi(k_z) \left[-\frac{k_z}{\omega\mu_2} T e^{ik_{2x}x} \right] e^{ik_z z},$$

$$H_{2z} = \int_{-\infty}^{+\infty} dk_z \phi(k_z) \left[\frac{k_{2x}}{\omega\mu_2} T e^{ik_{2x}x} \right] e^{ik_z z}, \tag{18}$$

in region 2. The unknown coefficients R, T, A_2, and B_2 possess similar meanings and expressions only except that we interchange the corresponding components μ_x and k_z (k_{1z}, k_{2z}) for the corresponding components of μ_z and k_x (k_{1x}, k_{2x}). For both cases, the time-averaged flux density can be expressed as

$$|\langle \vec{S}_i \rangle| = \frac{1}{2} \sqrt{[\mathrm{Re}(E_{iy} H_{iz}^*)]^2 + [\mathrm{Re}(E_{iy} H_{ix}^*)]^2}, \tag{19}$$

where the asterisk $(*)$ stands for the conjugation of the components of the magnetic field. This expression acts on the three different regions we considered. To observe the beam

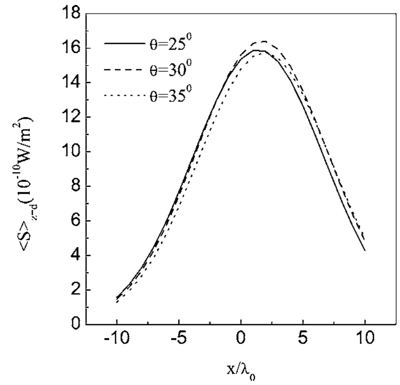

FIG. 2. Time-averaged flux density along x at $z=d$ for $\theta=25°,30°,35°$ incidence of a Gaussian beam. The solid ($\theta=25°$), and dashed ($\theta=30°$), and dash-dotted ($\theta=35°$) lines indicate the positive beam shifting for an anisotropic negative refractive medium.

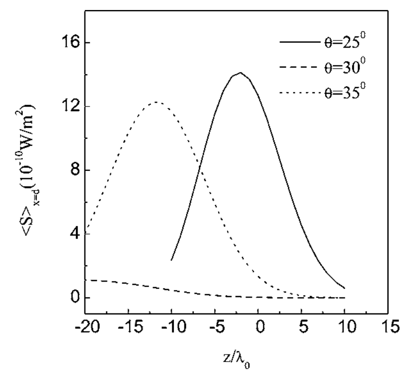

FIG. 3. Time-averaged flux density along z at $x=d$ for $\theta=25°,30°,35°$ incidence of a Gaussian beam. The solid ($\theta=25°$), dashed ($\theta=30°$), and dash-dotted ($\theta=35°$) lines indicate the negative beam shifting for an anisotropic negative refractive medium. The values of the solid line are scaled by 10^{-7}.

shifting, we focus our attention on the time-averaged flux density in region 2.

III. NUMERICAL RESULTS AND DISCUSSION

The numerical computation of the time-averaged flux density in region 2 has been performed by taking the concrete parameter values. $\varepsilon_1=\mu_1=1$, $\varepsilon_2=\mu_2=1$, $\omega=10^{10}$ Hz, incident angle $\theta=35°$, and $g=2$ are taken throughout. The thickness of slab is $d=4\lambda_0$, where λ_0 stands for the wavelength in vacuum. In order to verify the expressions described above, the isotropic positive refractive medium ($\varepsilon_x=\varepsilon_y=\varepsilon_z=1$ and $\mu_x=\mu_y=\mu_z=1$) and negative refractive medium ($\varepsilon_x=\varepsilon_y=\varepsilon_z=-1$ and $\mu_x=\mu_y=\mu_z=-1$) were computed and compared with Fig. 5 in Ref. [15]. Our results fit well with theirs. We also obtain that the beam shifting is positive for an isotropic positive refractive medium; on the contrary, the beam shifting is negative for an isotropic negative refractive medium which verifies that our expressions described are correct. Such results can be understood by considering the direction of energy flow in an isotropic positive or negative refractive medium. The tangential directions of the Poynting vector are parallel at the interface between two isotropic positive refractive media, which causes positive beam shifting naturally. Meanwhile, the tangential directions of the Poynting vector are antiparallel at the interface between isotropic positive and negative refractive media, which leads to negative beam shifting consequently. So what really counts is the direction of the tangential direction of the Poynting vector, which is the critical factor that determines whether the beam shifting is negative or not. Then we calculate the beam shiftings for our two cases by taking the following parameters used in Ref. [19]: $\varepsilon_x=\varepsilon_z=1$, $\varepsilon_y=-0.5$, $\mu_y=\mu_z=1$, and $\mu_x=-0.5$. We take the thickness of the slab as $d=4\lambda_0$. The influence of the incident angles on the time-averaged flux density against x and z is considered based on Eq. (19). The results are shown in Figs. 2 and 3 with three different incident angles $\theta=25°,30°,35°$, respectively. It is obvious that the beam shifting is positive for the first case (i.e., the interface is at the x-y plane), whereas it is negative for the second case (i.e., the interface is at the y-z plane). It is not difficult for us to understand such different results distinct from the case of an isotropic negative refractive slab. It is well known that the phase velocity and Poynting vector are antiparallel strictly in the isotropic negative refractive medium, which reveals the group refractive angle (determined by the Poynting vector) and phase refractive angle to be the same and negative. The negative group refractive angle will result in negative beam shifting naturally. However, in an anisotropic negative refractive medium, the phase velocity and Poynting vector are not antiparallel strictly [18]. Here isofrequency curves may be used to provide the refractive properties at the interface between two media conveniently. The question thus naturally arises in the wave vector plot: what is the relationship between the group velocity and the Poynting vector? If the dispersion relation is $\omega=\omega(\vec{k})$, the definition of the group velocity is $\vec{v}_g=\nabla_{\vec{k}}\omega(\vec{k})$, which means that the group velocity is a vector perpendicular to the normal surface of the isofrequency contour. The electric and magnetic fields in anisotropic material are described by $D_i=\varepsilon_{ik}E_k$ and $B_i=\mu_{ik}H_k$. For convenience, we express $\vec{k}=(\omega/c)\vec{n}$, where $\vec{n}$ is a vector which has the same direction of $\vec{k}$. The time average flux of energy in an electromagnetic field is expressed by $\vec{S}=\frac{1}{2}[\hat{\varepsilon}^{-1}\cdot(\vec{n}\times\vec{H})]\times[\hat{\mu}^{-1}\cdot(\vec{n}\times\vec{E})]$. Assume the vector $\vec{n}$ changes an infinitesimal amount $\delta\vec{n}$; the electric field and magnetic field coincide with the corresponding changes of $\delta\vec{E}$ and $\delta\vec{B}$. After carrying out some vector calculations, we obtain $\vec{E}\delta\vec{D}=[\vec{E}\vec{H}]\delta\vec{n}+\vec{B}\delta\vec{H}$ and $\vec{H}\delta\vec{B}=[\vec{E}\vec{H}]\delta\vec{n}+\vec{D}\delta\vec{E}$, where square brackets denote the cross product of the vectors. We would like to note that the relations $\vec{D}\delta\vec{E}=\varepsilon_{ik}E_k\delta E_i=\vec{E}\delta\vec{D}$ and $\vec{H}\delta\vec{B}=\mu_{ik}H_k\delta H_i=\vec{B}\delta\vec{H}$ can be satisfied if the permittivity and permeability are symmetric. Then we get $[\vec{E}\vec{H}]\delta\vec{n}=0$. Since $\delta\vec{n}$ is an arbitrary vector, we conclude that the direction of the Poynting vector is perpendicular to the plane of the wave vector—i.e., the isofrequency contour. Therefore, in our model, the group velocity and the Poynting vector have the same direction. We plot the wave vector diagrams for two different cases in Figs. 4 and

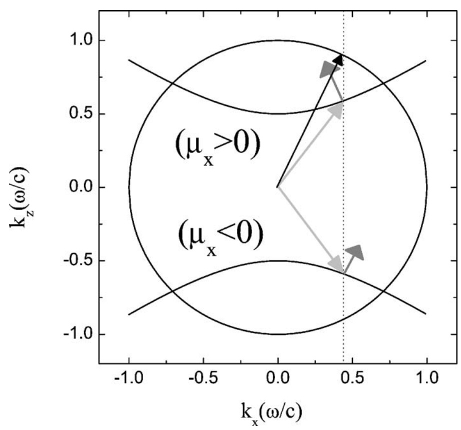

FIG. 4. Wave vector diagrams for the case that the interface between vacuum and an anisotropic medium is at the *x*-*y* plane. The circle and hyperbolic isofrequency curve correspond to the dispersion relations in the vacuum and anisotropic negative refractive medium, respectively. The black arrow stands for the incident wave vector. Two light gray arrows, which represent two possible k_z solutions according to the sign of μ_x. The direction of the gray arrows indicates the direction of the group velocity.

5. One is relating to the case that the interface between the vacuum and an anisotropic medium is at the *x*-*y* plane and the other is corresponding to the case that the interface is at the *y*-*z* plane. In Fig. 4, the abscissa and ordinate refer to the wave vector of k_x and k_z, respectively. Both of them are scaled by ω/c. The circle and hyperbolic isofrequency curve correspond to the dispersion relations in the vacuum and anisotropic negative refractive medium, respectively. We use a black arrow to stand for the incident wave vector. Due to the continuity of the wave vector along the *x* direction, there exist two light gray arrows, which represent two possible k_z solutions according to the different sign of μ_x. We know that

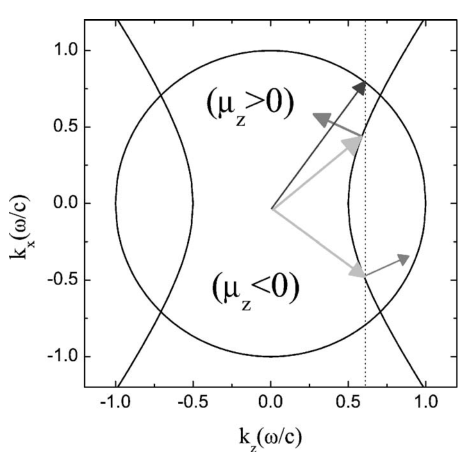

FIG. 5. Wave vector diagrams for the case that the interface between vacuum and an anisotropic medium is at the *y*-*z* plane. The circle and hyperbolic isofrequency curve correspond to the dispersion relations in the vacuum and anisotropic negative refractive medium, respectively. The black arrow stands for the incident wave vector. Two light gray arrows, which represent two possible k_x solutions according to the sign of μ_z. The direction of the gray arrows indicates the direction of the group velocity.

the direction of the group velocity $[\vec{v}_g=\nabla_{\vec{k}}\omega(\vec{k})]$ always lies normal to the isofrequency contour [16]. The calculation of the gradient gives the direction of increasing ω and thus provides the definite group velocity directions. Thus the group velocity directions shown in Figs. 4 and 5 are consonant with the direction of increasing ω. The components of Poynting vector from the incident medium are positive; the direction of the group velocity of anisotropic negative refractive material must be chosen to ensure the component of the Poynting vector along the *z* direction to be positive, as shown in Fig. 4. The tangential direction of the Poynting vector will determine whether the group refractive angle is positive or not. Then we get the conclusion that when μ_z is positive the group refractive angle is positive and when μ_z is negative the group refractive angle is negative. In Fig. 5, the abscissa and ordinate refer to the wave vector of k_z and k_x, respectively. Both of them are also scaled by ω/c. The circle and hyperbolic isofrequency curve also correspond to the dispersion relations in the vacuum and anisotropic negative refractive medium, respectively. The black arrow represents the incident wave vector. Different from the above case, now the wave vector along the *z* direction is continuous; there also exist two light gray arrows, which describe two possible k_x solutions according to the different sign of μ_z. The direction of the group velocity of anisotropic negative refractive material must be chose to ensure the component of the Poynting vector along the *x* direction to be positive, as shown in Fig. 5. We conclude that when μ_x is positive the group refractive angle is positive and when μ_x is negative the group refractive angle is negative. In our numerical calculation, we have negative μ_x and positive μ_z, which reveal a positive group refractive angle when the interface is at the *x*-*y* plane and a negative group refractive angle when the interface is at the *y*-*z* plane. Therefore for the first case that the interface is at the *x*-*y* plane, the group refractive angle is positive not only to ensure the Poynting vector inside the anisotropic negative refractive medium to point away from the interface, but also leads to a positive beam shifting naturally. On the contrary, the group refractive angle is negative to ensure not to violate the physics causality for the second case—i.e., the interface is at the *y*-*z* plane—and consequently bring out a negative beam shifting.

In order to investigate how the beam shifting evolves into the negative index material, we calculate the beam shiftings by taking the following parameter values as $\mu_x=-0.5$, $\mu_z=-1$ for the first case and $\mu_x=0.5$, $\mu_z=1$ for the second case. We plot the time-averaged flux density versus x/λ_0 and z/λ_0 in Figs. 6 and 7 at a fixed incident angle $\theta=35°$. The results show that the sign of μ_z determines the sign of the group refractive angle when the interface is at the *x*-*y* plane, while, when the interface is at the *y*-*z* plane, the positive or negative beam shifting is determined by the sign of μ_x, which is consistent with our theoretical discussions. A metamaterial composed of a two-dimensional periodic array of copper splitring resonators and wires was investigated by Ran *et al.* [13]. They examined the property of the material by simply measuring whether the beam shift is larger or smaller than some maximum shift and indeed found that the composite exhibits a left-handed behavior in a specific band. Further investiga-

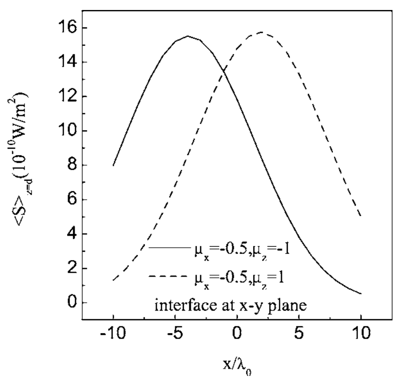

FIG. 6. Time-averaged flux density along the x direction at $z=d$ for an incidence of a Gaussian beam with $\theta=35°$. The solid line indicates the negative beam shifting for an anisotropic negative refractive medium when μ_z is negative; the dashed line indicates the positive beam shifting for an anisotropic negative refractive medium when μ_z is positive.

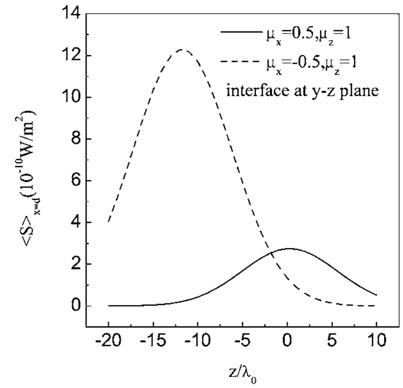

FIG. 7. Time-averaged flux density along z at $x=d$ for an incidence of a Gaussian beam with $\theta=35°$. The solid line indicates the positive beam shifting for an anisotropic negative refractive medium when μ_x is positive; the dashed line indicates the negative beam shifting for an anisotropic negative refractive medium when μ_x is negative.

tions of anisotropic negative refractive material, both theoretical and experimental, are definitely called for.

IV. CONCLUSIONS

The presence of a negative refractive index has led to unusual and unexplored phenomena in wave propagation. There exist different characters between conventional media and negative refractive media. Some brief discussions of beams shifting (Goos-Hänchen shifts) have been extended toward the case of nonabsorbing or weak absorbing isotropic negative refractive media [20,21]. Berman [22] has investigated the beam shifting in isotropic negative refractive media. Lakhtakia [23] has found that if the real part of the permittivity (permeability) of isotropic media is negative, only a $p(s)$-polarized beam experiences negative Goos-Hänchen shifts, and if the real part of both the permittivity and permeability is negative, the Goos-Hänchen shifts of p and s-polarized beams are negative for weak lossy negative refractive media. In this paper, we center on the beams shifting in anisotropic negative refractive media. Electric and magnetic fields inside and outside a slab made of an anisotropic negative refractive medium have been derived rigorously by using Maxwell's equations and coefficients determined from boundary conditions. We present a detailed analysis of the beam shifting from a Gaussian beam for an anisotropic negative refractive slab. Interestingly, we find that there exist two possible (positive or negative) beam shiftings for an anisotropic negative index medium due to the anisotropic properties, which is distinct from the case of an isotropic negative refractive index slab. The direction of the tangential component of the Poynting vector plays an important role in determining whether the beam shifting is negative (positive) or not. Applying the wave vector diagrams, we discuss the relations among the incident wave vector, reflective wave vector, and Poynting vector detailedly. For the first case that the interface is at the x-y plane, the sign of μ_x will decide the sign of the wave vector along the z direction and the sign of μ_z will determine the positive or negative of the group refractive angle, which leads to the positive or negative beam shifting directly. On the contrary, when the interface is at the y-z plane, the sign of μ_z will decide the sign of the wave vector along the x direction and the sign of μ_x will determine the positive or negative of the group refractive angle. Our calculation results indeed demonstrate these rules. Furthermore, our results show that the phase velocity and Poynting vector are not antiparallel strictly in an anisotropic negative refractive medium. It is the direction of the tangential component of the Poynting vector that determines whether the beam shifting is negative or positive, which helps us to understand the anisotropic negative refractive medium further.

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